

# Package ‘EigenR’

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**Type** Package

**Title** Complex Matrix Algebra with 'Eigen'

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**Description** Matrix algebra using the 'Eigen' C++ library: determinant, rank, inverse, pseudo-inverse, kernel and image, QR decomposition, Cholesky decomposition, Schur decomposition, Hessenberg decomposition, linear least-squares problems. Also provides matrix functions such as exponential, logarithm, power, sine and cosine. Complex matrices are supported.

**License** GPL-3

**URL** <https://github.com/stla/EigenR>

**BugReports** <https://github.com/stla/EigenR/issues>

**Depends** R (>= 3.0.2)

**Imports** Rcpp (>= 1.0.5)

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Eigen_absdet	<i>Absolute value of the determinant</i>
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---

### Description

Absolute value of the determinant of a real matrix.

### Usage

Eigen\_absdet(M)

### Arguments

M                    a *real* square matrix

### Value

The absolute value of the determinant of M.

**Note**

'Eigen\_absdet(M)' is not faster than 'abs(Eigen\_det(M))'.

**Examples**

```
set.seed(666L)
M <- matrix(rpois(25L, 1), 5L, 5L)
Eigen_absdet(M)
```

---

 Eigen\_chol

*Cholesky decomposition of a matrix*


---

**Description**

Cholesky decomposition of a symmetric or Hermitian matrix.

**Usage**

```
Eigen_chol(M)
```

**Arguments**

M a square symmetric/Hermitian positive-definite matrix or [SparseMatrix](#), real/complex

**Details**

Symmetry is not checked; only the lower triangular part of M is used.

**Value**

The upper triangular factor of the Cholesky decomposition of M.

**Examples**

```
M <- rbind(c(5,1), c(1,3))
U <- Eigen_chol(M)
t(U) %*% U # this is `M`
# a Hermitian example:
A <- rbind(c(1,1i), c(1i,2))
( M <- A %*% t(Conj(A)) )
try(chol(M)) # fails
U <- Eigen_chol(M)
t(Conj(U)) %*% U # this is `M`
# a sparse example
M <- asSparseMatrix(diag(1:5))
Eigen_chol(M)
```

---

Eigen\_complexSchur      *Complex Schur decomposition*

---

**Description**

Complex Schur decomposition of a square matrix.

**Usage**

```
Eigen_complexSchur(M)
```

**Arguments**

M                      real or complex square matrix

**Details**

See [Eigen::ComplexSchur](#).

**Value**

A list with the T and U matrices.

**Examples**

```
library(EigenR)
M <- cbind(c(3, 2i, 1+3i), c(1, 1i, 1), c(5, 0, -2i))
schur <- Eigen_complexSchur(M)
T <- schur$T
U <- schur$U
M - U %**% T %**% t(Conj(U))
```

---

Eigen\_cos                      *Matrix cosine*

---

**Description**

Matrix cosine of a real or complex square matrix.

**Usage**

```
Eigen_cos(M)
```

**Arguments**

M                      a square matrix, real or complex

**Value**

The matrix cosine of M.

**Examples**

```
library(EigenR)
M <- toeplitz(c(1,2,3))
cosM <- Eigen_cos(M)
sinM <- Eigen_sin(M)
cosM %% cosM + sinM %% sinM # identity matrix
```

---

Eigen\_cosh

*Matrix hyperbolic cosine*

---

**Description**

Matrix hyperbolic cosine of a real or complex square matrix.

**Usage**

```
Eigen_cosh(M)
```

**Arguments**

M                    a square matrix, real or complex

**Value**

The matrix hyperbolic cosine of M.

**Examples**

```
library(EigenR)
M <- toeplitz(c(1,2,3))
Eigen_cosh(M)
(Eigen_exp(M) + Eigen_exp(-M)) / 2 # identical
```

---

Eigen\_det                      *Determinant of a matrix*

---

**Description**

Determinant of a real or complex matrix.

**Usage**

```
Eigen_det(M)
```

**Arguments**

M                      a square matrix or [SparseMatrix](#), real or complex

**Value**

The determinant of M.

**Examples**

```
set.seed(666)
M <- matrix(rpois(25, 1), 5L, 5L)
Eigen_det(M)
# determinants of complex matrices are supported:
Eigen_det(M + 1i * M)
# as well as determinants of sparse matrices:
Eigen_det(asSparseMatrix(M))
Eigen_det(asSparseMatrix(M + 1i * M))
```

---

Eigen\_exp                      *Exponential of a matrix*

---

**Description**

Exponential of a real or complex square matrix.

**Usage**

```
Eigen_exp(M)
```

**Arguments**

M                      a square matrix, real or complex

**Value**

The exponential of M.

---

Eigen_Hessenberg	<i>Hessenberg decomposition</i>
------------------	---------------------------------

---

**Description**

Hessenberg decomposition of a square matrix.

**Usage**

```
Eigen_Hessenberg(M)
```

**Arguments**

M                    real or complex square matrix

**Details**

See [Eigen::HessenbergDecomposition](#).

**Value**

A list with the H and Q matrices.

**Examples**

```
library(EigenR)
M <- cbind(c(3, 2i, 1+3i), c(1, 1i, 1), c(5, 0, -2i))
Eigen_Hessenberg(M)
```

---

Eigen_inverse	<i>Inverse of a matrix</i>
---------------	----------------------------

---

**Description**

Inverse of a real or complex matrix.

**Usage**

```
Eigen_inverse(M)
```

**Arguments**

M                    an invertible square matrix, real or complex

**Value**

The inverse matrix of M.

---

Eigen\_isInjective      *Check injectivity*

---

**Description**

Checks whether a matrix represents an injective linear map (i.e. has trivial kernel).

**Usage**

```
Eigen_isInjective(M)
```

**Arguments**

M                      a matrix, real or complex

**Value**

A Boolean value indicating whether M represents an injective linear map.

**Examples**

```
set.seed(666L)
M <- matrix(rpois(35L, 1), 5L, 7L)
Eigen_isInjective(M)
```

---

Eigen\_isInvertible      *Check invertibility*

---

**Description**

Checks whether a matrix is invertible.

**Usage**

```
Eigen_isInvertible(M)
```

**Arguments**

M                      a matrix, real or complex

**Value**

A Boolean value indicating whether M is invertible.

**Examples**

```
set.seed(666L)
M <- matrix(rpois(25L, 1), 5L, 5L)
Eigen_isInvertible(M)
```



---

Eigen\_isSurjective      *Check surjectivity*

---

**Description**

Checks whether a matrix represents a surjective linear map.

**Usage**

```
Eigen_isSurjective(M)
```

**Arguments**

M                      a matrix, real or complex

**Value**

A Boolean value indicating whether M represents a surjective linear map.

**Examples**

```
set.seed(666L)
M <- matrix(rpois(35L, 1), 7L, 5L)
Eigen_isSurjective(M)
```

---

Eigen\_kernel              *Kernel of a matrix*

---

**Description**

Kernel (null-space) of a real or complex matrix.

**Usage**

```
Eigen_kernel(M, method = "COD")
```

**Arguments**

M                      a matrix, real or complex  
method                one of "COD" or "LU"; the faster method depends on the size of the matrix

**Value**

A basis of the kernel of M. With method = "COD", the basis is orthonormal, while it is not with method = "LU".

**See Also**

[Eigen\\_kernelDimension](#).

**Examples**

```
set.seed(666)
M <- matrix(rgamma(30L, 12, 1), 10L, 3L)
M <- cbind(M, M[,1]+M[,2], M[,2]+2*M[,3])
# basis of the kernel of `M`:
Eigen_kernel(M, method = "LU")
# orthonormal basis of the kernel of `M`:
Eigen_kernel(M, method = "COD")
```

---

`Eigen_kernelDimension` *Dimension of kernel*

---

**Description**

Dimension of the kernel of a matrix.

**Usage**

```
Eigen_kernelDimension(M)
```

**Arguments**

M                    a matrix, real or complex

**Value**

An integer, the dimension of the kernel of M.

**See Also**

[Eigen\\_isInjective](#), [Eigen\\_kernel](#).

**Examples**

```
set.seed(666L)
M <- matrix(rpois(35L, 1), 5L, 7L)
Eigen_kernelDimension(M)
```

---

Eigen_log	<i>Logarithm of a matrix</i>
-----------	------------------------------

---

**Description**

Logarithm of a real or complex square matrix, when possible.

**Usage**

Eigen\_log(M)

**Arguments**

M                    a square matrix, real or complex

**Details**

The logarithm of a matrix does not always exist. See [matrix logarithm](#).

**Value**

The logarithm of M.

---

Eigen_logabsdet	<i>Logarithm of the absolute value of the determinant</i>
-----------------	---

---

**Description**

Logarithm of the absolute value of the determinant of a real matrix.

**Usage**

Eigen\_logabsdet(M)

**Arguments**

M                    a *real* square matrix

**Value**

The logarithm of the absolute value of the determinant of M.

**Note**

‘Eigen\_logabsdet(M)’ is not faster than ‘log(abs(Eigen\_det(M)))’.

**Examples**

```
set.seed(666L)
M <- matrix(rpois(25L, 1), 5L, 5L)
Eigen_logabsdet(M)
```

---

**Eigen\_lsSolve***Linear least-squares problems*

---

**Description**

Solves a linear least-squares problem.

**Usage**

```
Eigen_lsSolve(A, b, method = "cod")
```

**Arguments**

A	a $n \times p$ matrix, real or complex
b	a vector of length $n$ or a matrix with $n$ rows, real or complex
method	the method used to solve the problem, either "svd" (based on the SVD decomposition) or "cod" (based on the complete orthogonal decomposition)

**Value**

The solution  $X$  of the least-squares problem  $AX \approx b$  (similar to `lm.fit(A, b)$coefficients`). This is a matrix if  $b$  is a matrix, or a vector if  $b$  is a vector.

**Examples**

```
set.seed(129)
n <- 7; p <- 2
A <- matrix(rnorm(n * p), n, p)
b <- rnorm(n)
lsfit <- Eigen_lsSolve(A, b)
b - A %*% lsfit # residuals
```

---

Eigen_pinverse	<i>Pseudo-inverse of a matrix</i>
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---

**Description**

Pseudo-inverse of a real or complex matrix (Moore-Penrose generalized inverse).

**Usage**

```
Eigen_pinverse(M)
```

**Arguments**

M                    a matrix, real or complex, not necessarily square

**Value**

The pseudo-inverse matrix of M.

**Examples**

```
library(EigenR)
M <- rbind(
  toeplitz(c(3, 2, 1)),
  toeplitz(c(4, 5, 6))
)
Mplus <- Eigen_pinverse(M)
all.equal(M, M %*% Mplus %*% M)
all.equal(Mplus, Mplus %*% M %*% Mplus)
#' a complex matrix
A <- M + 1i * M[, c(3L, 2L, 1L)]
Aplus <- Eigen_pinverse(A)
Aplus <- A %*% Aplus
all.equal(Aplus, t(Conj(Aplus))) #' `A %*% Aplus` is Hermitian
AplusA <- Aplus %*% A
all.equal(AplusA, t(Conj(AplusA))) #' `Aplus %*% A` is Hermitian
```

---

Eigen_pow	<i>Matricial power</i>
-----------	------------------------

---

**Description**

Matricial power of a real or complex square matrix, when possible.

**Usage**

```
Eigen_pow(M, p)
```

**Arguments**

M                    a square matrix, real or complex  
p                    a number, real or complex, the power exponent

**Details**

The power is defined with the help of the exponential and the logarithm. See [matrix power](#).

**Value**

The matrix M raised at the power p.

---

Eigen_QR	<i>QR decomposition of a matrix</i>
----------	-------------------------------------

---

**Description**

QR decomposition of a real or complex matrix.

**Usage**

```
Eigen_QR(M)
```

**Arguments**

M                    a matrix, real or complex

**Value**

A list with the Q matrix and the R matrix.

**Examples**

```
M <- cbind(c(1,2,3), c(4,5,6))  
x <- Eigen_QR(M)  
x$Q %*% x$R
```

---

Eigen_range	<i>Range of a matrix</i>
-------------	--------------------------

---

**Description**

Range (column-space, image, span) of a real or complex matrix.

**Usage**

```
Eigen_range(M, method = "QR")
```

**Arguments**

M	a matrix, real or complex
method	one of "LU", "QR", or "COD"; the "LU" method is faster

**Value**

A basis of the range of M. With method = "LU", the basis is not orthonormal, while it is with method = "QR" and method = "COD".

---

Eigen_rank	<i>Rank of a matrix</i>
------------	-------------------------

---

**Description**

Rank of a real or complex matrix.

**Usage**

```
Eigen_rank(M)
```

**Arguments**

M	a matrix, real or complex
---	---------------------------

**Value**

The rank of M.

---

Eigen\_realSchur      *Real Schur decomposition*

---

**Description**

Real Schur decomposition of a square matrix.

**Usage**

```
Eigen_realSchur(M)
```

**Arguments**

M                      real square matrix

**Details**

See [Eigen::RealSchur](#).

**Value**

A list with the T and U matrices.

**Examples**

```
library(EigenR)
M <- cbind(c(3, 2, 3), c(1, 1, 1), c(5, 0, -2))
schur <- Eigen_realSchur(M)
T <- schur$T
U <- schur$U
M - U %*% T %*% t(U)
```

---

Eigen\_sin              *Matrix sine*

---

**Description**

Matrix sine of a real or complex square matrix.

**Usage**

```
Eigen_sin(M)
```

**Arguments**

M                      a square matrix, real or complex



**Value**

The matrix sine of M.

---

Eigen_sinh	<i>Matrix hyperbolic sine</i>
------------	-------------------------------

---

**Description**

Matrix hyperbolic sine of a real or complex square matrix.

**Usage**

```
Eigen_sinh(M)
```

**Arguments**

M a square matrix, real or complex

**Value**

The matrix hyperbolic sine of M.

**Examples**

```
library(EigenR)
M <- toeplitz(c(1,2,3))
Eigen_sinh(M)
(Eigen_exp(M) - Eigen_exp(-M)) / 2 # identical
```

---

Eigen_sqrt	<i>Square root of a matrix</i>
------------	--------------------------------

---

**Description**

Square root of a real or complex square matrix, when possible.

**Usage**

```
Eigen_sqrt(M)
```

**Arguments**

M a square matrix, real or complex

**Details**

See [matrix square root](#).

**Value**

A square root of M.

**Examples**

```
# Rotation matrix over 60 degrees:
M <- cbind(c(cos(pi/3), sin(pi/3)), c(-sin(pi/3), cos(pi/3)))
# Its square root, the rotation matrix over 30 degrees:
Eigen_sqrt(M)
```

---

Eigen\_UtDU

*'UtDU' decomposition of a matrix*

---

**Description**

Cholesky-'UtDU' decomposition of a symmetric or Hermitian matrix.

**Usage**

```
Eigen_UtDU(M)
```

**Arguments**

M a square symmetric/Hermitian positive or negative semidefinite matrix, real/complex

**Details**

Symmetry is not checked; only the lower triangular part of M is used.

**Value**

The Cholesky-'UtDU' decomposition of M in a list (see example).

**Examples**

```
x <- matrix(c(1:5, (1:5)^2), 5, 2)
x <- cbind(x, x[, 1] + 3*x[, 2])
M <- crossprod(x)
UtDU <- Eigen_UtDU(M)
U <- UtDU$U
D <- UtDU$D
perm <- UtDU$perm
UP <- U[, perm]
t(UP) %*% diag(D) %*% UP # this is `M`
```

---

SparseMatrix	<i>Sparse matrix</i>
--------------	----------------------

---

**Description**

Constructs a sparse matrix, real or complex.

**Usage**

```
SparseMatrix(i, j, Mij, nrows, ncols)
```

```
## S3 method for class 'SparseMatrix'  
print(x, ...)
```

```
asSparseMatrix(M)
```

**Arguments**

<code>i, j</code>	indices of the non-zero coefficients
<code>Mij</code>	values of the non-zero coefficients; must be a vector of the same length as <code>i</code> and <code>j</code> or a single number which will be recycled
<code>nrows, ncols</code>	dimensions of the matrix
<code>x</code>	a <code>SparseMatrix</code> object
<code>...</code>	ignored
<code>M</code>	a matrix, real or complex

**Value**

A list with the class `SparseMatrix`.

**Examples**

```
set.seed(666)  
( M <- matrix(rpois(50L, 1), 10L, 5L) )  
asSparseMatrix(M)
```

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