

# Package ‘gmresls’

January 17, 2025

**Title** Solve Least Squares with GMRES(k)

**Version** 0.2.3

**Date** 2025-01-17

**Description** Solves a least squares system  $Ax \sim b$  ( $\dim(A)=(m,n)$  with  $m \geq n$ ) with a precondition matrix  $B$ :  $B Ax = B b$  ( $\dim(B)=(n,m)$ ). Implemented method is based on GMRES (Saad, Youcef; Schultz, Martin H. (1986). ``GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems" <[doi:10.1137/0907058](https://doi.org/10.1137/0907058)>) with callback functions, i.e. no explicit  $A$ ,  $B$  or  $b$  are required.

**License** GPL ( $\geq 3$ )

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**Suggests** RUnit

**BugReports** <https://forgemia.inra.fr/mathscell/gmresls/-/issues>

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2025-01-17 15:20:01 UTC

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gmresls

*Solve a Least Squares System with a Preconditioner.***Description**

Solve a least squares system  $Ax \approx b$  ( $\dim(A)=(m,n)$  with  $m \geq n$ ) with a preconditioner  $B$ :  $BAx=Bb$  ( $\dim(B)=(n,m)$ ). Implemented method uses GMRES(k) with callback functions, i.e. no explicit  $A$  or  $B$  are required. GMRES can be restarted after  $k$  iterations.

**Usage**

```
gmresls(f_resid, f_BAx, x0 = NULL, k = 0, maxit = 0, tol = 1e-07, ...)
```

**Arguments**

<code>f_resid</code>	A function <code>f_resid(x, ...)</code> calculating $B(b-Ax)$ for a given $x$ . If $x$ is of length 0 (e.g. <code>NULL</code> ), it must be considered as 0. If $x_0$ is not <code>NULL</code> in the call to <code>gmres()</code> then $x$ won't be <code>NULL</code> in the <code>f_resid(x, ...)</code> call either.
<code>f_BAx</code>	A function <code>f_BAx(x, ...)</code> calculating matrix-matrix-vector product $BAx$ for a given $x$ .
<code>x0</code>	A vector or <code>NULL</code> (which means 0), initial approximation for $Ax=b$
<code>k</code>	An integer, parameter for restarting GMRES. Value 0 (default) means no restart, i.e. at most $\text{length}(x)$ basis vectors will be constructed and used.
<code>maxit</code>	A maximal iteration number. Here, iteration number continues to increment even after a possible GMRES restart. Default (0) means $\text{length}(x)$ .
<code>tol</code>	A tolerance for solution $x$ , estimated as $\ B(Ax-b)\ /\ Bb\ $ , default $1.e-7$
<code>...</code>	Parameters passed through to <code>f_BAx</code> and <code>f_resid</code>

**Details**

Implemented method is equivalent to a classical GMRES(k) method with restart after constructing  $k$  basis vectors and applied to a square system  $BAx=Bb$ . Dense matrices constructed and stored by this method are of size  $(\text{length}(x), k)$  and  $(k+1, k)$  where  $k$  is GMRES current basis vector number. If  $\text{maxit} > k$ , GMRES will be restarted after each  $k$  iterations. Particularity of this implementation that matrices  $A$  and  $B$  have no to be stored explicitly. User provides just callback function mimicking their multiplication by adequate vectors. In case of non convergence after  $\text{maxit}$  iterations, `attr(x)` will contain a field 'warning' with the message which will be also issued with `warning()`. If the operator  $BA$  is not of full rank, iterations will be stopped before reaching convergence or  $\text{maxit}$ . A warning will be emitted in this case.

**Value**

The solution  $x$ , having the structure of `f_resid(x,...)`.

**Examples**

```
# prepare a 4x3 toy least squares (LS) problem Ax=b
A=rbind(diag(1:3)+matrix(1, 3,3), rep(1, 3))
xsol=1:3
b=A%%xsol+rnorm(4, 0., 0.1) # add some noise as it is often the case in LS
f_resid=function(x,...)
  with(list(...), if (length(x) == 0L) crossprod(A, b) else crossprod(A, b-A%%x))
f_BAx=function(x,...)
  with(list(...), crossprod(A, A%%x))
x=gmresls(f_resid, f_BAx, A=A, b=b)
stopifnot(all.equal(c(x), c(qr.solve(A,b))))
```

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